

## Practice Makes Perfect Understanding

### **Problem 1:**

$f$  is a linear function. Values of  $x$  and  $f(x)$  are given in the table below; complete the table.

$x$	$f(x)$
-3	17
0	--
--	1
4	-18
7	--
--	-30

### **Problem 2:**

A family of linear functions is given by

$$f(x) = mx + (3 - 2m)$$

where  $x$  is the independent variable and  $m$  is a constant.

- Graph  $f$  for  $m = 0, 1, 2, -3$  and  $-5$
- What do all the graphs in part a) have in common?
- Justify your answer to part b) analytically.
- Write the equation of the family of functions whose graphs pass by the same point  $(-2, -4)$ .

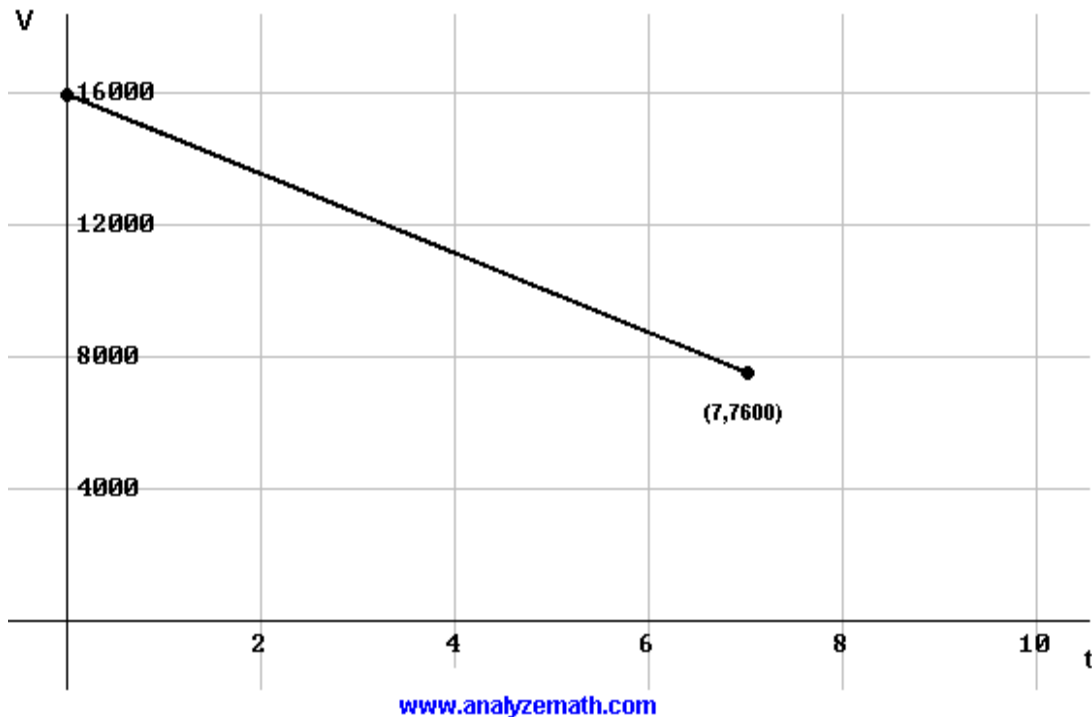
### **Problem 3:**

A high school had 1200 students enrolled in 2003 and 1500 students in 2006. If the student population  $P$ ; grows as a linear function of time  $t$ , where  $t$  is the number of years after 2003.

- How many students will be enrolled in the school in 2010?
- Find a linear function that relates the student population to the time  $t$ .

### **Problem 4:**

The graph shown below is that of the linear function that relates the value  $V$  (in \$) of a car to its age  $t$ , where  $t$  is the number of years after 2000.



- a) Find the slope and interpret it.
- b) What will be the value of the car in the year 2010?

### Answer sheet

#### Solution to Problem 1:

- $f$  is a linear function whose formula has the form

$$f(x) = a x + b$$

- where  $a$  and  $b$  are constants to be found. Note that 2 ordered pairs  $(-3, 17)$  and  $(4, -18)$  are given in the table. These two ordered pairs are used to write a system of linear equations as follows

$$17 = -3 a + b \text{ and } -18 = 4 a + b$$

- Solve the above system to obtain  $a = -5$  and  $b = 2$  and write the formula for function  $f$  as follows

$$f(x) = -5 x + 2$$

- We now use the formula for  $f$  to find  $f(x)$  given  $x$  or find  $x$  given  $f(x)$ .

for  $x = 0$  ,  $f(0) = -5(0) + 2 = 2$

for  $f(x) = 1$  ,  $1 = -5x + 2$  which gives  $x = 1/5$

for  $x = 7$  ,  $f(7) = -5(7) + 2 = -33$

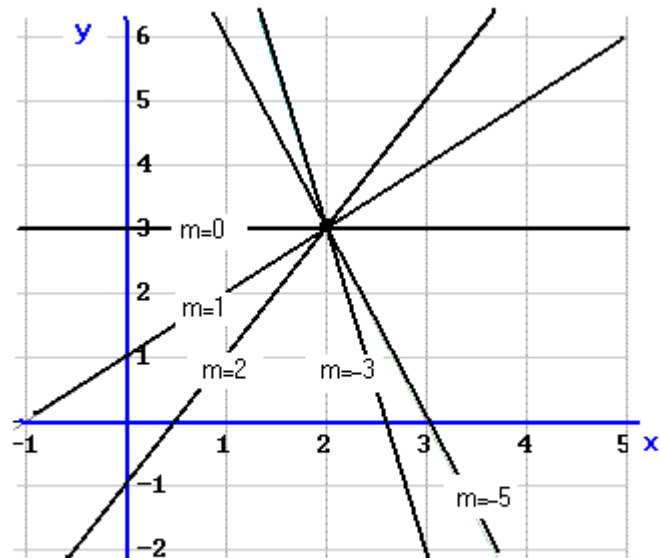
for  $f(x) = -30$  ,  $-30 = -5x + 2$  which gives  $x = 32/5$

- We now put the values calculated above in the table.

$x$	$f(x)$
-3	17
0	2
1/5	1
4	-18
7	-33
32/5	-30

### Solution to Problem 2:

- a)



- b) All the graphs pass by the same point (2 , 3)
- c) To prove that all lines described by the equation  $f(x) = mx + (3 - 2 m)$  pass by the point (2 , 3), show that  $f(2) = 3$

$$f(2) = 2 m + (3 - 2m) = 3$$

- d) The point slope form of the equation of a line is used to find equation of the family of lines that pass by the point (-2,-4) is found as follows

$$y - (-4) = m (x - (-2))$$

$$y = mx + (2m - 4)$$

- As an exercise, graph the above equation for different values of m and check that all the lines obtained pass by the point (-2 , -4)

### **Solution to Problem 3:**

- a) The given information may be written as ordered pairs (t , P). The year 2003 correspond to t = 0 and the year 2006 corresponds to t = 3, hence the 2 ordered pairs

$$(0, 1200) \text{ and } (3, 1500)$$

- Since the population grows linearly with the time t, we use the two ordered pairs to find the slope m of the graph of P as follows

$$m = (1500 - 1200) / (6 - 3) = 100 \text{ students / year}$$

- The slope  $m = 100$  means that the students population grows by 100 students every year. From 2003 to 2010 there are 7 years and the students population in 2010 will be

$$P(2010) = P(2003) + 7 * 100 = 1200 + 700 = 1900 \text{ students.}$$

- b) We know the slope and two points, we may use the point slope form to find an equation for the population P as a function of t as follows

$$P - P_1 = m (t - t_1)$$

$$P - 1200 = 100 (t - 0)$$

$$P = 100 t + 1200$$

#### **Solution to Problem 4:**

- a) Find two points from the graph

(0 , 16000) and (7 , 7600)

- Use the above points to find the slope m

$$m = (7600 - 16000) / (7 - 0) = - 1200 \$ / \text{year}$$

- A slope of - 1200 \$ / year means that the value of the car decreases by 1200 \$ every year.
- b) In 2010,  $t = 10$ . There are 3 years from  $t = 7$  to  $t = 10$ . The value of the car will be given by

$$7600 - 3 * 1200 = 4000 \$$$

[http://www.analyzemath.com/math\\_problems/linear\\_func\\_problems.html](http://www.analyzemath.com/math_problems/linear_func_problems.html)